Giant Resonances in ^{94}Mo

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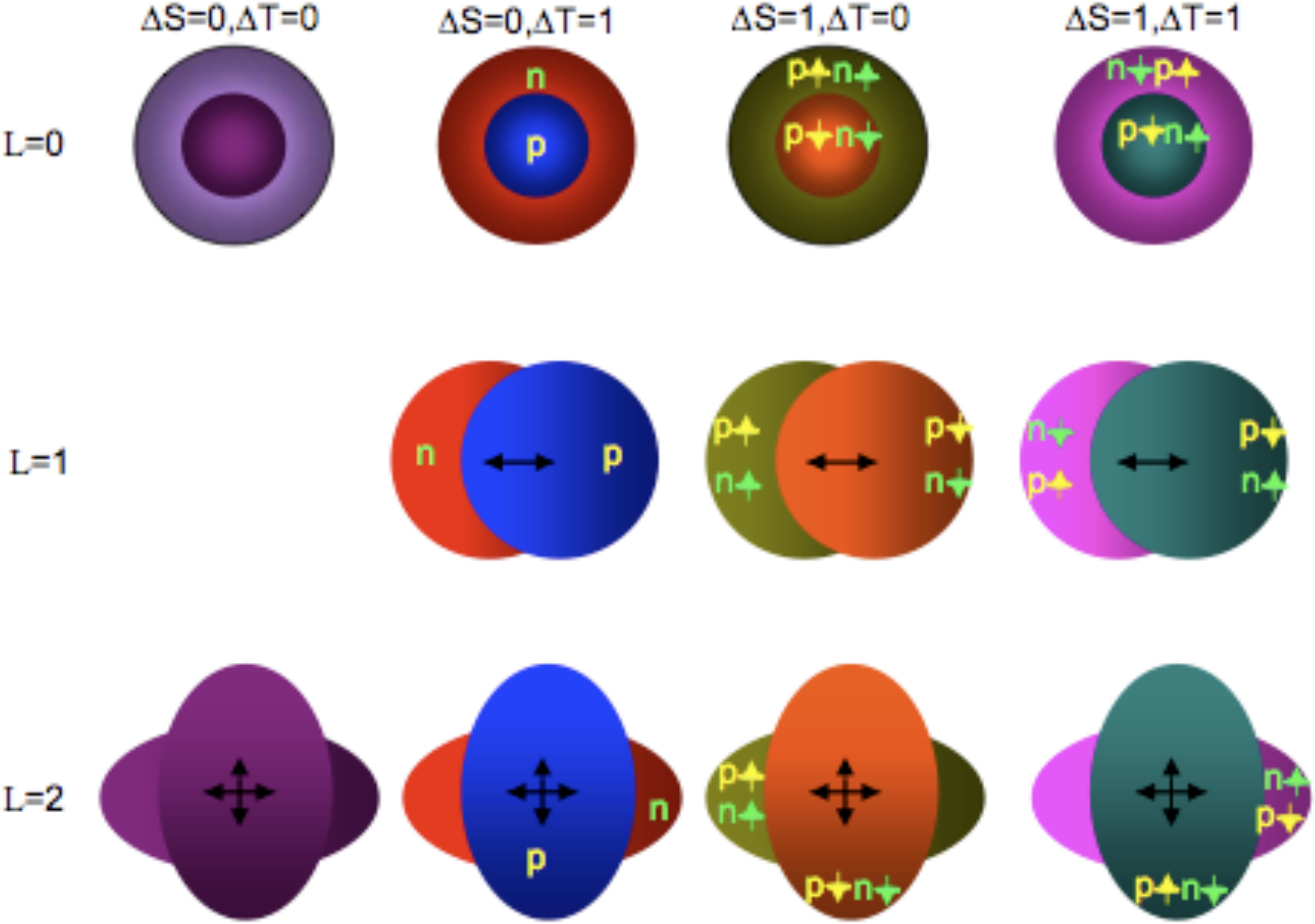
Giant Resonances

- Highly collective states in nuclei that are discovered by scattering experiments
- Observed at energies between 10 and 40 MeV
- Classified by multipolarity L, isospin T, and spin S
- L=0 is the monopole, L=1 dipole, L=2 quadrupole, and L=3 octupole.
- T=0 state is an isoscalar, and T=1 is the isovector. This describes the phase difference between the neutrons' and protons' oscillations.
- S is the spin difference in the nucleon oscillations.
- In this work, we are concerned with Isoscalar electric (S=0) multipoles. Most important is the Isoscalar Giant Monopole Resonance (ISGMR).



S. S. Hanna, in Proceedings of the Giant Multipole Resonance Topical Conference, edited by F. E. Bertrand (Harwood Academic publisher, Oak Ridge, Tennessee, 1979)

Equation of State (EOS) of Nuclear Matter

- The EOS of nuclear matter (NM) is the binding energy of NM per nucleon as a function of the density $E[\rho]$.
- EOS are important for the study of nuclear astrophysics, nuclear reactions, and nuclear structure.
- Some properties of the EOS can be determined by studying Giant Resonances.

- EOS of symmetric Nuclear Matter (NM) is $E_0[\rho]$. It integrates the energy density per nucleon $e_0(\rho)$ and maps nuclear density ρ to energy per nucleon.
- $E_0[\rho]$ has a minimum at the saturation density $\rho_0=\sim .16\ fm^{-3}$ with a binding energy per nucleon of 16 MeV.

Nonsymmetric NM

- NM is not always symmetric. More generally, $E[\rho_p, \rho_n]$ maps proton density ρ_p and neutron density ρ_n to energy per nucleon.
- This can be written as a combination of E_0 and E_{sym}, the symmetry energy.

$$E[\ _p,\ _n] = E_0[\ ] + E_{sym}[\ ]\ \frac{\ _n \qquad \ _p}{\ }^{2}$$

- We can Taylor expand these functions around ρ_o.

NM Properties

$$E[\ _p, \ _n] = E_0[\ ] + E_{sym}[\ ]\ \frac{\ _n \quad \ _p}{\ }\ ^2$$

$$E_0[\ ] \quad E[\ _0] + \frac{1}{18}K \ \frac{\ _0}{\ _0}\ ^2$$

$$E_{sym}[\ ] \quad J + \frac{1}{3}L \ \frac{\ _n \quad \ _p}{\ _0} + \frac{1}{18}K_{sym} \ \frac{\ _n \quad \ _p}{\ _0}\ ^2$$

$$K = 9 \ ^2 \frac{d^2E_0[\ ]}{d \ ^2}\bigg|_0$$

- The coefficients of expansion are defined to be various NM properties [1].
- The ISGMR is sensitive to K, the incompressibility coefficient of nuclear matter.

$$J = E_{sym}[\ _0]$$

$$L = 3 \ \frac{dE_{sym}[\ ]}{d}\bigg|_0$$

$$K_{sym} = 9 \ ^2 \frac{d^2E_{sym}[\ ]}{d \ ^2}\bigg|_0$$

Hartree-Fock Method (HF)

- Solving the many-body Schrödinger equation $\hat{H} \quad = E$ is difficult. Here the Hamiltonian is

$$\hat{H} = {}_{i=1}^{A} \frac{\hat{p}^2}{2m} + {}_{i<j}^{A} \hat{V}_{ij}.$$

- Approximations must be used to obtain a solution.
- We assume that the total nuclear wave function can be written as a Slater determinant of single particle wave functions $_i$ [4].

$$(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n) = \frac{1}{\sqrt{n!}} \begin{vmatrix} {}_1(\bar{x}_1) & \cdots & {}_1(\bar{x}_n) \\ \vdots & \ddots & \vdots \\ {}_n(\bar{x}_1) & \cdots & {}_n(\bar{x}_n) \end{vmatrix}$$

- The ground state is obtained by minimizing the Hamiltonian expectation value:

$$\langle \quad | \hat{H} | \quad \rangle.$$

The HF Equations

- Vary the single particle wave functions and set the variation of the expectation value to zero.

$$ {}_i \qquad {}_i{}^{+} \qquad {}_i $$

$$\frac{\hbar^2}{2m} \;{}^2\; {}_i(\mathbf{r}) + \;{}_i(\mathbf{r})U_H(\mathbf{r}) \quad d^3\mathbf{r}'U_F(\mathbf{r},\mathbf{r}') \;{}_i(\mathbf{r}') = e_i \;{}_i(\mathbf{r})$$

$$U_H(\mathbf{r}) = \;{}_{j=1}^{A}\; d^3\mathbf{r}' \;{}_j^{*}(\mathbf{r}')V(\mathbf{r},\mathbf{r}') \;{}_j(\mathbf{r}') \qquad U_F(\mathbf{r},\mathbf{r}') = \;{}_{j=1}^{A}\; {}_j^{*}(\mathbf{r}')V(\mathbf{r},\mathbf{r}') \;{}_j(\mathbf{r})$$

- The HF equations are nonlinear and solved iteratively.

Random Phase Approximation (RPA)

- Macroscopically, Giant resonances are described as liquid oscillations.
- Microscopically, we can view them as a linear combination of particle-hole excitations.
- RPA is an approximation that the giant resonances are a linear combination of particle-hole excitations from the ground state.

Skyrme Interactions

- Skyrme interactions are momentum dependent two body interactions with 10 parameters. They do not include the Coulomb interaction.
- Contact interaction $(\mathbf{r} \quad \mathbf{r}')$
- $t_{i,}$ $\mathrm{x}_{i,}$ $\quad , W_0$ are the parameters.

$$V_{ij} = t_0(1 + x_0 P_{ij} \;) \; (\mathbf{r}_i \quad \mathbf{r}_j) + \frac{1}{2} t_1(1 + x_1 P_{ij} \;) \; \overleftarrow{k}_{ij}^2 \; (\mathbf{r}_i \quad \mathbf{r}_j) + \; (\mathbf{r}_i \quad \mathbf{r}_j)\overrightarrow{k}_{ij}^2 \quad +$$

$$t_2(1 + x_2 P_{ij} \;)\overleftarrow{k}_{ij} \; (\mathbf{r}_i \quad \mathbf{r}_j)\overrightarrow{k}_{ij} + \frac{1}{6} t_3(1 + x_3 P_{ij} \;) \quad \frac{\mathbf{r}_i + \mathbf{r}_j}{2} \quad (\mathbf{r}_i \quad \mathbf{r}_j) + iW_0 \overleftarrow{k}_{ij} \left(\vec{\;}_i + \vec{\;}_j \right) \; (\mathbf{r}_i \quad \mathbf{r}_j)\overrightarrow{k}_{ij}$$

Skyrme Interactions

- Hundreds sets of parameters .
- With 10 parameters, there are many different local minimums that can be found while fitting.
- Each one predicts different sets of nuclear matter properties and energy spectra.
- We used 33 common Skyrme interactions found in [3].

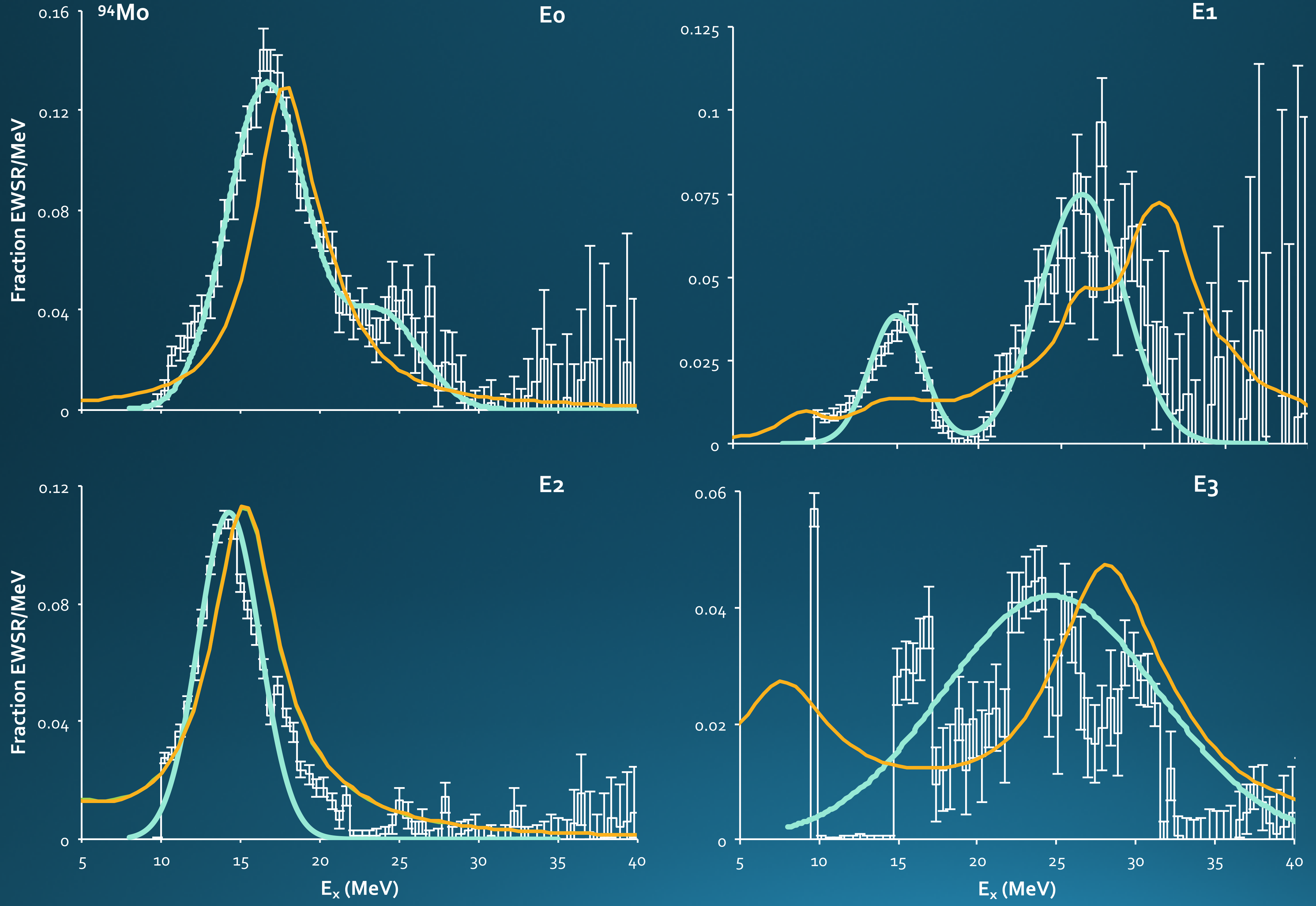
Strength Functions

- Strength functions S(E) measure scattering transitions.
- For a scattering operator F_L,

$$S(E) = \quad {}_{j} \left|\langle 0|F_L|j\rangle\right|^2 \quad (E_j \quad E_0).$$

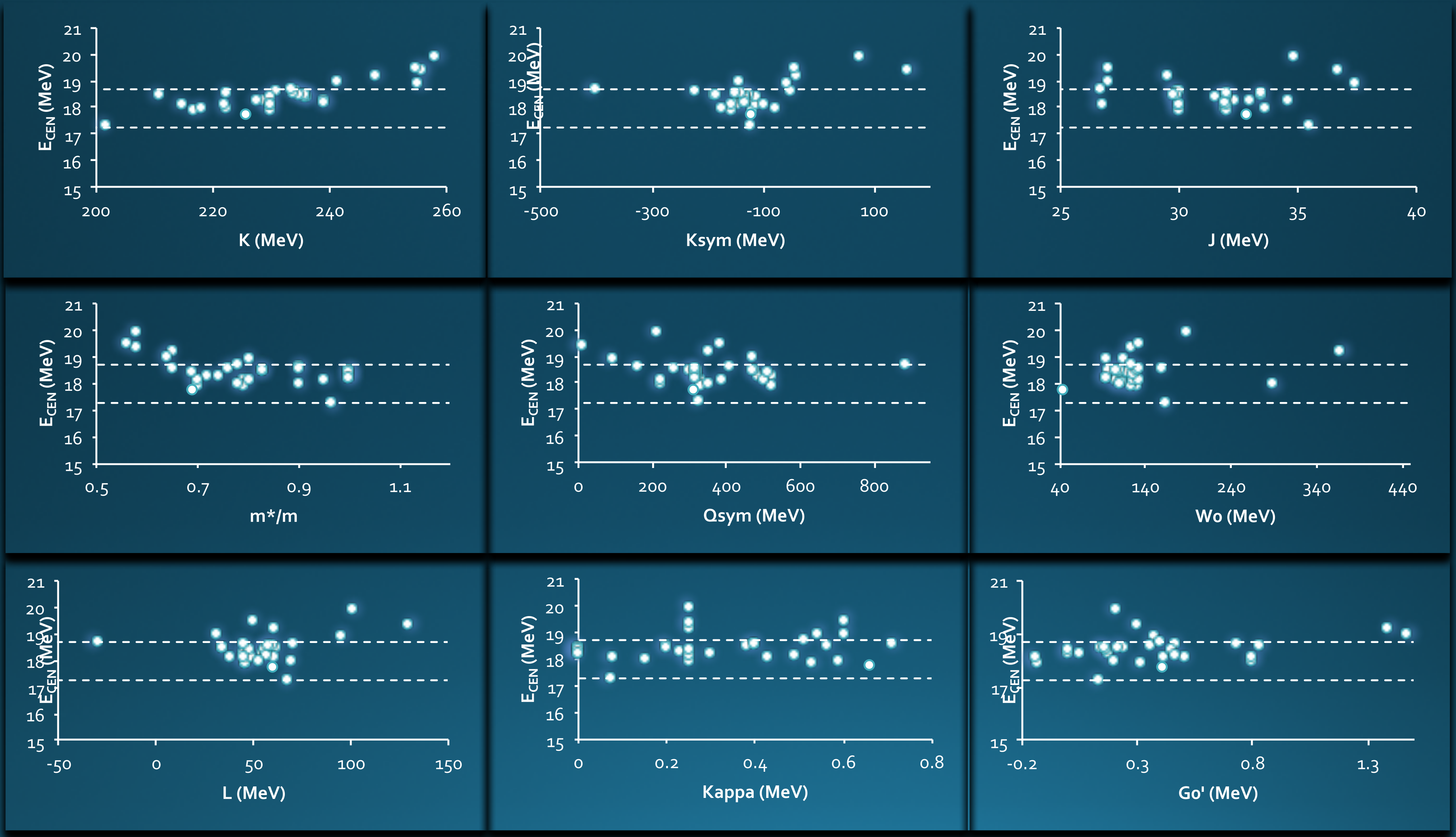
- $|0\rangle$ is the HF RPA ground state and the sum over j is over all HF RPA states. The form of these scattering operators for the different multipoles can be found in [1].
- We define energy moments m_k and the centroid energy E_{cen}.

$$m_k = \quad {}_{0} \; dE \; E^k S(E) \qquad\qquad E_{cen} = \frac{m_1}{m_0}$$

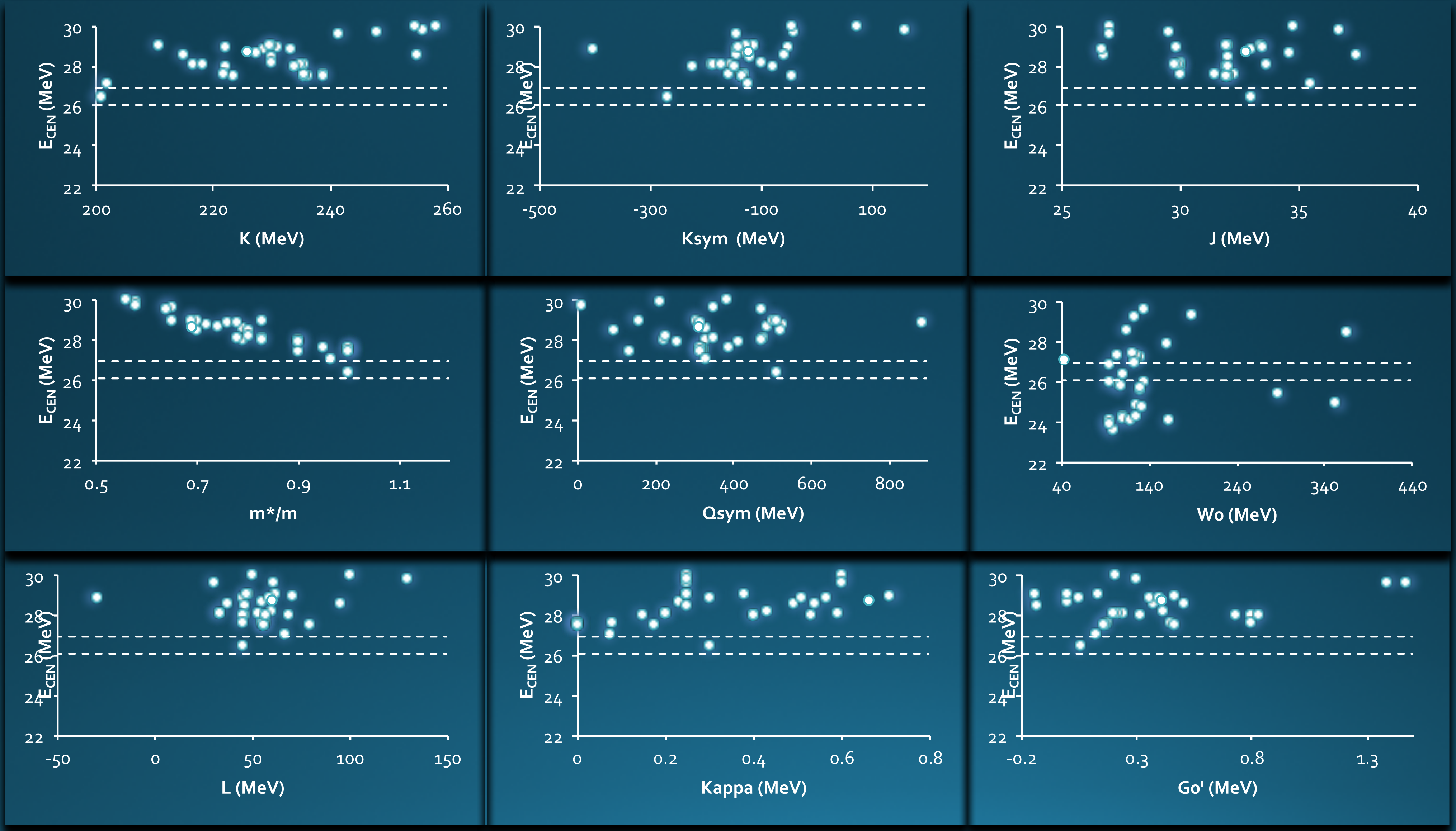


The Gaussian fits (light blue) and the KDE0 calculations (yellow) graphed together. The data is from [2].

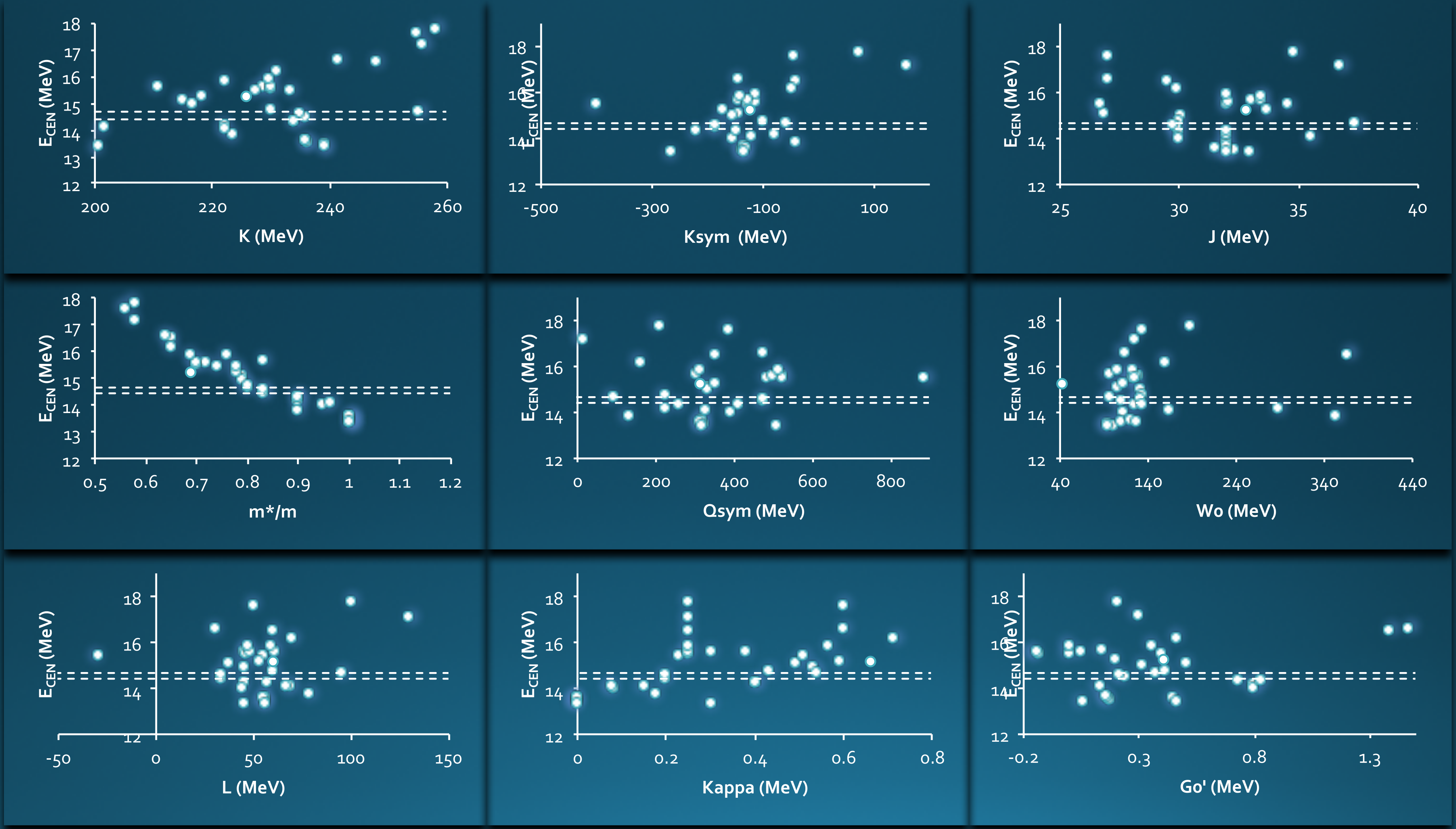
^{94}Mo L0


E_{CEN} (MeV)
K (MeV)
Ksym (MeV)
J (MeV)
m*/m
Qsym (MeV)
Wo (MeV)
L (MeV)
Kappa (MeV)
Go' (MeV)

^{94}Mo L1 high peak

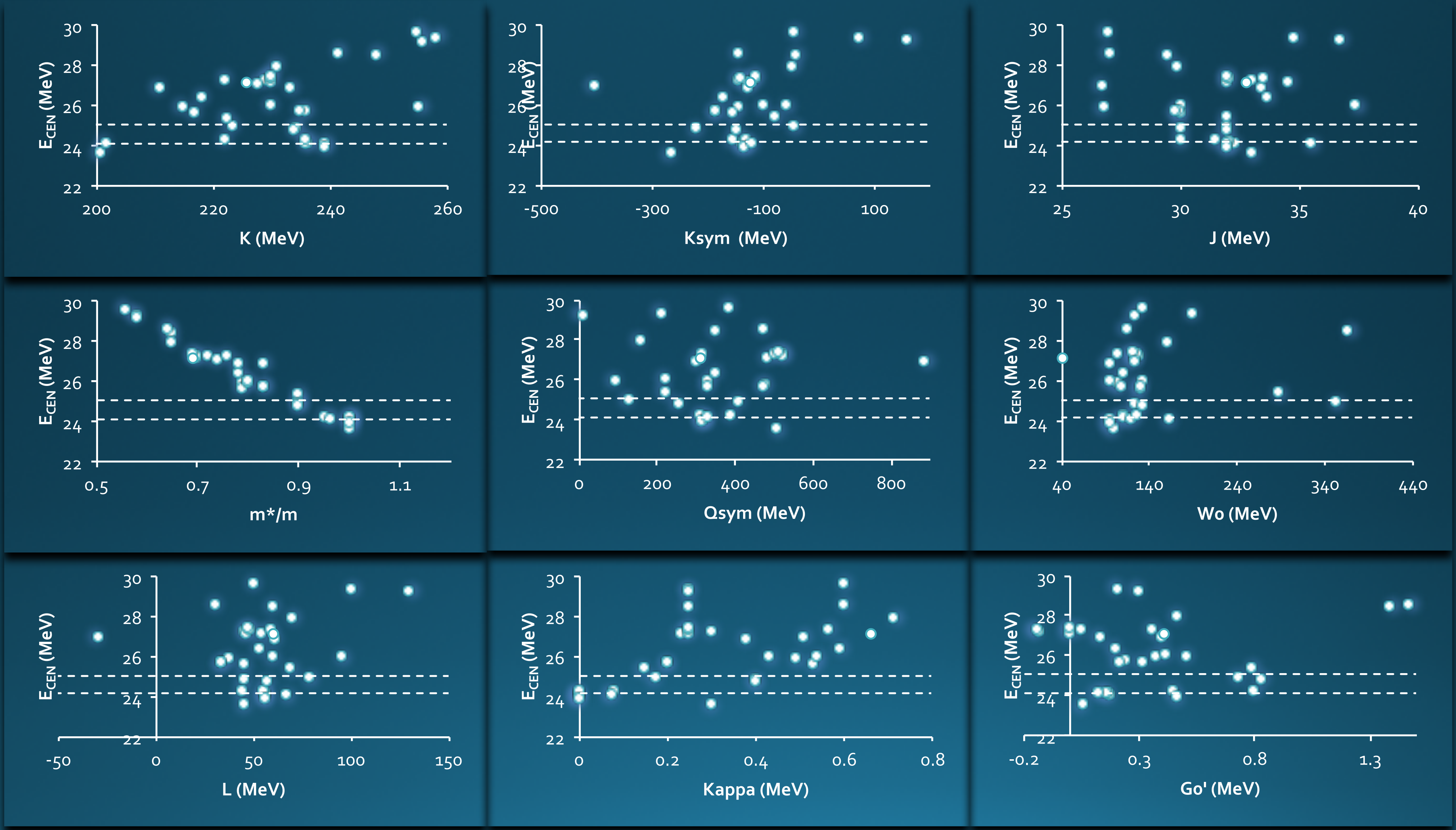


^{94}Mo L2



E_CEN (MeV)
K (MeV)
Ksym (MeV)
J (MeV)
m*/m
Qsym (MeV)
Wo (MeV)
L (MeV)
Kappa (MeV)
Go' (MeV)

^{94}Mo L3


E_CEN (MeV)
K (MeV)
Ksym (MeV)
J (MeV)
m*/m
Qsym (MeV)
Wo (MeV)
L (MeV)
Kappa (MeV)
Go' (MeV)

Conclusions

- These results are consistent with the currently accepted value of $K = 240 \pm 20$ MeV and the effective mass $m^*/m = 0.8 \pm 0.1$.
- There is a second peak observed in E0 that is not predicted by our model.

Acknowledgements

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References

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[2] J. Button, et al. PRC. To be published,

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